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Model-Based Feedforward Precompensation and VS-Type Robust Nonlinear Postcompensation for Uncertain Robotic Systems with/without Knowledge of Uncertainty Bounds (Ⅱ)

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In this paper, the robust nonlinear controller for an uncertain robot system is developed and characterized with a unified method. Based on deterministic approach, the control structure consists of two parts : In the first part, the primary control law is synthesized to precompensate for the nominal system ; and in the second part the adaptive version of robust controllers are utilized to postcompensate for the system uncertainties. The uncertainties assumed in this papar are bounded by higher-order polynomials in the Euclidean norms of system states without knowledge of bounding coefficients. Using the Lyapunov stability theory, we can guarantee that all possible responses of the closed-loop system are at least uniformly and ultimately bounded. The tracking properties of the control algorithms are verified through numerical simulations, and the results show that the proposed controllers are proven to be robust enough for any higher-order system uncertainty.

Key Words : Higher-order Uncertainties, Uncertainty Bounds, Trajectory Tracking, Adaptive Law.

1. Introduction

Robotics and automation technologies provide increased productivity in flexible manufacturing and services industries. Thus there has been a considerable interest in the design of high-performance and reliable control algorithms for robotic manipulators. It is recognized that a class of nonlinear dynamical systems contain various uncertainties in which the system uncertainties can be divided into model-parametric (or structured) uncertainties and unstructured uncertainties. Since most of present control schemes often ignore the uncertainties while robots in motion, these algorithms cannot be used in a wide range of operating conditions.

Among the various approaches to the control of robotic manipulators, model-based controls have been presented. Feedforward compensation and computed torque techniques are well known examples of the model-based method. Early researches on this approach (without system uncertainties) were performed by Craig et al. and among others. However, under significant uncertainties, the tracking performance of the robot system will be significantly impaired and may even become unstable. During the last decade of research, a great deal of effort has been expended on the control problem for dynamical systems in cases where physical models are not completely known.(Corless, 1983 ; Chen, 1990 ; Abdallah, 1991 ; Spong,1992 ; Shi, 1992 ; Qu, 1992 ; You, 1994a ; Ioannou, 1984 ; Ortega, 1989 ; Sadegh, 1990 ; You, 1994b) In this study, we introduce the control strategy for uncertain robot systems based on the deterministic approach.(Corless, 1983 ; Chen, 1990 ; Abdallah, 1991 ; Spong,1992 ; Shi,

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1992 ; Qu, 1992 ; You, 1994a ; Sadegh, 1990 ; You, 1994b) One useful design method for controlling uncertain systems is variable structure (VS) controls (Qu, 1992 ; You, 1994a ; You, 1994b) ; however, most of the controllers generally require a priori knowledge of uncertainty bounds. More recently, Abdallah et al. give a survey of robust controls for robotic manipulators. Other way to overcome model-parametric uncertainties is to use adaptive control methods. Up to this point, a considerable amount of study has been done on the adaptive control scheme. (Craig, 1987 ; Corless, 1983 ; Chen, 1990 ; Ioannou, 1984 ; Ortega, 1989 ; Sadegh, 1990) But to the authors' knowledge, the current studies rely heavily on estimating the model-parameters of system (i.e., centralized adaptive control). Ortega and Spong (Ortega, 1989) present a recent review of adaptive robot control. Instead, a decentralized control technique will be introduced in this work. To date, there have been few studies (Shi, 1992 ; You, 1994b) discussing the stability issues of the robust adaptive controller under higher-order system uncertainties, and several major problems still remain. Specifically, it has been shown that the control strategies, such as adaptive controls and robust controls, have the following shortcomings from the practical point of view : cope with relatively small uncertainties ; utilize computationally complex algorithms ; synthesize purely discontinuous controllers ; require a priori knowledge of the uncertainty bounds.

The purpose of the study is to further investigate and characterize the robust motion tracking control for uncertain robotic manipulators in which the system uncertainties are characterized deterministically. Additionally, the corresponding control laws will overcome all the defects found in earlier design. The control algorithms presented consists of two major components : model-based feedforward plus PD controller is first synthesized to stabilize the nominal system ; then the robust nonlinear control law is introduced to handle the system uncertainties. The uncertainties assumed here are bounded by higher-order polynomials in the norms of system states. Since no information on the uncertainty bounds is available, the adaptive bounds of the robust controllers are presented to directly update the unknown bounds on line. The present study offers clear evidence that the corresponding closed-loop systems are at least uniformly ultimately bounded under significant uncertainties.

This paper is organized as follows : Background and problem formulations are presented in Sec. 2. In Sec. 3, without possible knowledge of the uncertainties, the adaptive versions of the robust controllers are formulated. In Sec. 4, simulation results are presented and discussed. Finally, the conclusions of this work are summarized in Sec. 5.

2. Background and Problem Formulations

As noted in the previous paper (1), the dynamic model of an n-link rigid manipulator is compactly shown to be

$$\boldsymbol{M}(\boldsymbol{q};\Theta)\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}};\Theta)\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q};\Theta)+\boldsymbol{T}_u(\boldsymbol{q},\dot{\boldsymbol{q}})=\boldsymbol{T},\ \forall t\geq 0 \quad (1)$$

where $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, and $\ddot{\boldsymbol{q}}\in R^n$ are the joint position, velocity, and acceleration vectors, respectively ; $\boldsymbol{M}(\boldsymbol{q};\Theta)\in R^{n\times n}$ is the inertia matrix ; $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}};\Theta)\dot{\boldsymbol{q}}\in R^n$ is the vector of the Coriolis/centrifugal forces ; $\boldsymbol{G}(\boldsymbol{q};\Theta)\in R^n$ is the gravity force vector ; $\boldsymbol{T}\in R^n$ is the joint torque vector ; $T_u(\boldsymbol{q},\dot{\boldsymbol{q}})\in R^n$ represents the vector of the unstructured uncertainties, which is generally not considered in most of other control design ; Θ is an $m\times 1$ vector of robot parameters. Note that the physical meanings of all terms in Eq. (1) as well as its fundamental properties have been given previously in detail.

Now, some assumptions are made for the subsequent problem formulations.

[A1] : The unstructured uncertainties ($\boldsymbol{T}_u$) strongly influences system performance.
It is worth noting the fact that the structure of $\boldsymbol{T}_u$ assumed here will be bounded by higher-order polynomials in the system states rather than obeying either constant magnitude or first-order polynomials.

[A2] : The true parameter vector, $\Theta=$

$[\theta_1\theta_2\cdots\theta_m]^T$ is assumed to be unknown as the manipulator moves. In addition, the variation of θ_i is within the range $\Psi_i := [\underline{\theta}_i, \overline{\theta}_i] \subset R$, $\forall i \in [1, m]$, where $\underline{\theta}_i$ and $\overline{\theta}_i$ are the unknown positive constants. Therefore, we have $\Psi := \Psi_1 \times \cdots \times \Psi_m$ and $\Theta \in \Psi \subset R^m$ in which Ψ is particularly unknown nonempty set.

[A3] : The desired trajectory ($\boldsymbol{q}_d \in C^2$ function) and its derivatives are all continuous and uniformly bounded by

$$d_1 = \sup_{t \geq 0} \| \boldsymbol{q}_d \|, \; d_2 = \sup_{t \geq 0} \| \dot{\boldsymbol{q}}_d \| \text{ and}$$
$$d_3 = \sup_{t \geq 0} \| \ddot{\boldsymbol{q}}_d \|$$

where d_1, d_2 and d_3 ($< \infty$) are especially unknown positive constants.

From now on, a class of trajectory tracking errors are defined as follows : $\boldsymbol{e} \in R^n$ is the vector of position tracking errors ; $\boldsymbol{e}_p = \boldsymbol{q} - \boldsymbol{q}_d$, where $\boldsymbol{q}_d \in R^n$ is the desired position vector ; the reference tracking errors, $\dot{\boldsymbol{e}}_r \in R^n$, are defined by $\dot{\boldsymbol{e}}_r = \dot{\boldsymbol{q}}_d - \boldsymbol{F}\boldsymbol{e}_p$, with $\boldsymbol{F} = \varepsilon \boldsymbol{E}_n (\varepsilon > 0)$; the sliding variable vector, $\boldsymbol{e}_s \in R^n$, is defined as $\boldsymbol{e}_s = \dot{\boldsymbol{q}} - \dot{\boldsymbol{e}}_r = \dot{\boldsymbol{e}}_v + \boldsymbol{F}\boldsymbol{e}_p$.

Provided that some system states ($\boldsymbol{q}$, $\dot{\boldsymbol{q}}$) are available from measurements and also that the desired paths ($\boldsymbol{q}_d$, $\dot{\boldsymbol{q}}_d$, $\ddot{\boldsymbol{q}}_d$) are all smooth and bounded functions of time ($\in L_\infty$). Then this study describes a design methodology for trajectory tracking controllers, which reduces the sensitivity of the closed-loop system to uncertainties and assures the following system behavior : Given the system dynamics (1) with some or all robot parameters being unknown, we present the robust nonlinear control algorithm, $\boldsymbol{T} = \boldsymbol{h}(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d ; \Theta_0)$, $t \geq 0$, in such a way that every trajectory in the closed-loop system is kept within desirable tolerance $\overline{r}_p$ and $\overline{r}_v \in R^+$ (with smallest values possible). For example, the solutions of the corresponding closed-loop system are at least uniformly ultimately bounded under the significant uncertainties by (Corless, 1983 ; Chen, 1990 ; You, 1994c ; Sastry, 1989)

$$\limsup_{t \to \infty} \| \boldsymbol{e}_p(t) \| \leq \overline{r}_p \text{ and}$$
$$\limsup_{t \to \infty} \| \dot{\boldsymbol{e}}_v(t) \| \leq \overline{r}_v$$

where the tolerance can be considered as a measure of closeness of the system tracking errors to asymptotic (or exponential) stability (i.e., $\overline{r}_p = \overline{r}_v = 0$). And Θ_0 is the nominal (or fixed) values of Θ (in which Θ_0 and Θ are defined on the same sets), and $\boldsymbol{h}$ is the vector of nonlinear functions in R^n.

In designing a robust controller for an uncertain system, we generally need the knowledge of possible upper bounds on the uncertainties. Especially in deterministic method, robust control algorithms are developed by measuring size of uncertainties, which are typically characterized in the appropriate norms.

As mentioned above, the design objective is to formulate a class of control input vector so that the actual system responses track the desired quantities as closely and fast as possible. In this study, the general control structure (see Fig. 1) is given by (You, 1994a ; You, 1994b)

$$\boldsymbol{T} = \overline{\boldsymbol{T}}_m + \hat{\overline{\boldsymbol{T}}}_n, \; t \geq 0 \tag{2}$$

with

$$\overline{\boldsymbol{T}}_m = \boldsymbol{M}_0(\boldsymbol{q}_d ; \Theta_0)\ddot{\boldsymbol{e}}_r + \boldsymbol{C}_0(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d ; \Theta_0)\dot{\boldsymbol{e}}_r + \boldsymbol{G}_0(\boldsymbol{q}_d ; \Theta_0) - \boldsymbol{K}\boldsymbol{e}_s$$

in which $\boldsymbol{M}_0$, $\boldsymbol{C}_0$ and $\boldsymbol{G}_0$ denote the estimated versions (not via adaptation mechanism) of the true values $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{G}$, respectively ; the feedback gain matrix $\boldsymbol{K} = K\boldsymbol{E}_n$ is chosen by the system designer ($K > 0$). Thus the first part of the control law ($\overline{\boldsymbol{T}}_m$) consists of model-based feedforward compensation and PD feedback terms ; and the auxiliary control input is given by $\hat{\overline{\boldsymbol{T}}}_n = -\boldsymbol{f}(\boldsymbol{e}_p, \dot{\boldsymbol{e}}_v, \boldsymbol{e}_s)$, where $\boldsymbol{f} \in R^n$ are some nonlinear functions on ($\boldsymbol{e}_p$, $\dot{\boldsymbol{e}}_v$, $\boldsymbol{e}_s$), which will be specified later in details. An important aspect to realize is that the control algorithm under consideration is

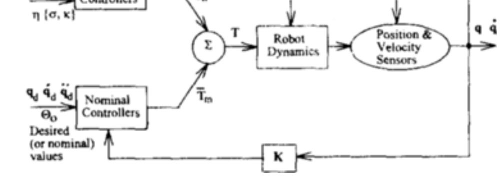

Fig. 1 Block diagram of the proposed control algorithm with the adaptive bounds of robust controllers

sum of two major parts: the primary control vector, $\overline{T}_m \in R^n$, is designed to stabilize the nominal system without the uncertainties, and the robust nonlinear control, $\overline{\overline{T}}_n \in R^n$, which is chosen to be a continuous or discontinuous function, is intended to account for both the compensation error (or modeling error) and unstructured uncertainties. This two-stage control scheme is intended to achieve better robustness as well as tracking performance to significant uncertainties. In the case that $M_0 = C_0 = G_0 = 0$ in (2), the control structure is simply reduced to $T = -Ke_s + \overline{\overline{T}}_n$. In addition, if $\overline{\overline{T}}_n = 0$, then $T = M_0 \ddot{e}_r + C_0 \dot{e}_r + G_0 - Ke_s$.

After some algebraic manipulations, the closed-loop error dynamics can be compactly expressed as

$$M(q; \Theta)\dot{e}_s = -C(q, \dot{q}; \Theta)e_s - (T_u + T_s) - Ke_s + \overline{\overline{T}}_n \quad (3)$$

where

$$T_s = [M - M_0]\ddot{e}_r + [C - C_0]\dot{e}_r + [G - G_0] \quad (4)$$

which represents the structured uncertainties due to model-parametric variations (or incorrect parameter values). In real systems, it should be noted that $T_s \neq 0$, that is, the estimated parameter values do not match the actual ones.

As stated earlier, the robust control approaches usually require the evaluation of possible upper bounds on the system uncertainties, which is shown in the paper (I). Suppose that the extended tracking error vector, $\overline{z} \in R^{2n}$, is defined as $\overline{z} = [e_p^T, \dot{e}_v^T]^T$. In what follows, we will give the possible upper bounds on the uncertainties.

Lemma 1: The structured uncertainties (T_s) are bounded in the form

$$\| T_s \| \leq a_0 + a_1 \| \overline{z} \| + a_2 \| \overline{z} \|^2 = \bar{\omega}_s$$

where a_i are unknown positive constants, which depend on the size of parametric variations and desired trajectories while the robot manipulator moves.

Proof: See the paper (I) for the complete proof.

[A4]: The unstructured uncertainties (T_u) are given as

$$\| T_u \| \leq b_0 + b_1 \| \overline{z} \| + \cdots + b_m \| \overline{z} \|^m = \sum_{i=0}^{m} b_i \| \overline{z} \|^i = \bar{\omega}_u$$

where b_i are unknown positive constants, and m is the highest order of $\bar{\omega}_u$ in the system uncertainties.

Based on the above observations, we can obtain the following result.

Lemma 2: The combined uncertainties are bounded by

$$\| T_s + T_u \| \leq \| T_s \| + \| T_u \| \leq c_0 + c_1 \| \overline{z} \| + \cdots + c_m \| \overline{z} \|^m = \sum_{i=0}^{m} c_i \| \overline{z} \|^i = \Phi_s$$

where c_i $(i = 0, \cdots, m)$ are especially unknown constants; thus, $\Phi_s (\in R^+)$ is a continuous and unknown scalar bounding function.

Proof: See the paper (I) for the detailed proof. As a result, the combined uncertainties are bounded by higher-order polynomials in the norms of the system states with the unknown coefficients $c \in R^{m+1}$.

In our studies, two robust design schemes are employed to postcompensate for the significant uncertainties in (3): The possible upper bounds on the uncertainties are assumed to be known for the nonadaptive robust control law ($\overline{T}_n$), as given in the paper (I). Without knowledge of the bounds, the adaptive bounds of the robust controllers ($\overline{\overline{T}}_n$) will be designed to directly estimate the uncertainty bounds in this paper (II), where the circumflex ($\hat{\bullet}$) represents the estimated version of ($\bullet$) by adaptive law.

3. Robust Controllers with Adaptive Uncertainty Bounds

In the previous paper (I), we studied the robust nonlinear controller based on *a priori* knowledge of uncertainty bounds. Generally, the system uncertainties are completely unknown, thus the least upper bounds may not be easily obtained nor feasible to draw. In this section and throughout the rest of the paper, by relaxing prerequisite on the uncertainties, we concisely extend the results of the paper (I) to the control method with adaptive uncertainty bounds. This methodology is called a decentralized robust adaptive controller, which combines VS-type robust control and adaptive control techniques.

The structural properties of the uncertainty bounds are given previously, i.e.,

$$\| T_s \| + \| T_u \| \le \Phi_s$$

On the other hand, the vector of uncertainty bound coefficients, c (or Φ_s), is completely unknown rather than being assumed known. For developing adaptive mechanism on the uncertainty bounds, we define new functions $U_s^T \in R^{m+1}$ as $U_s := [1 \; \| \bar{z} \| \cdots \| \bar{z} \|^m]$, which is referred to as the "regressor-like function".(You, 1994c) And the unknown coefficient vector $c \in R^{m+1}$ is given as

$$c = [c_0 \; c_1 \; \cdots \; c_m]^T, \; c_i \in R^+$$

Hence the uncertainty bounds in Lemma 2 are further expressed as

$$\textstyle\sum_{i=0}^{m} c_i \| \bar{z} \|^i = \Phi_s = U_s c$$

which represents the linear parameterization of the unknown bounding function. Moreover, the estimated version via adaptive law is

$$\hat{\Phi}_s(t, e_p, \dot{e}_v) = U_s \hat{c}(t)$$

where $\hat{c}(t) = [\hat{c}_0 \cdots \hat{c}_m]^T$ is the estimated vector of c. Now, the vector of uncertainty bound errors, $\tilde{c} \in R^{m-1}$, can be defined by $\tilde{c}(t) = c - \hat{c}(t)$. Then, we choose a modified adaptation algorithm (You, 1994b) to update the unknown gains online:

$$\dot{\hat{c}}(t) = P_s(U_s^T \| e_s \| - \omega_s \hat{c}) \tag{5}$$

where the adaptation gain $P_s \in R^{(m+1)\times(m+1)}$ may be selected as a positive-definite matrix. It is important to note that the fixed leakage term ω_s (>0) belongs to σ-modification law,(Chen, 1990; Ioannou, 1984; You, 1994b) which is intended to provide the robustness against the uncertainties.

To ensure the aforementioned design objective, the robust postcompensation with adaptive bound is given by (see Fig.1)

$$\bar{\bar{T}}_n = -\frac{\hat{\Phi}_s^2 e_s}{\| e_s \| \hat{\Phi}_s + \eta}, \; t \ge 0 \tag{6}$$

with $\eta(t) = \sigma \exp(-\kappa t)$, where the free parameters (σ and κ) are non-negative constants. In this algorithm, a conservative functional structure on Φ_s may be assumed to deal with any higher-order uncertainties. In case of $\eta = 0$ (or $\sigma = 0$), the control action becomes a purely VS-type control law, which is discontinuous on the surface $e_s = 0$. In practice, it causes undesirable phenomena, such as chattering associated with excessive control activity and exciting high-frequency unmodelled dynamics. Now, we provide the main results in the following theorem.

Theorem: With the unknown constants c on the uncertainty bounds, the solutions of the closed-loop system (3) with (5) and (6) are at least uniformly ultimately bounded; that is, every solution starting in H^c approaches the compact (closed and bounded) set H, and thereafter remains for all future time in H:

$$H = \{(e_s, \tilde{c}) \in R^n \times R^{(m+1)} : \; V \le \bar{V}_f\}$$

in which

$$\eta \ne 0 (\sigma > 0, \; \kappa = 0); \; \bar{V}_f = (\sigma + \xi)/\xi_0$$

and

$$\eta \ne 0 (\sigma > 0, \; \kappa = 0); \; \bar{V}_f = \begin{cases} \xi/\xi_0, & \xi_0 \ne \kappa \\ \xi, & \xi_0 = \kappa \end{cases}$$

with $\xi = (\omega_s/2) \| c \|^2$ and $\xi_0 = \min\{2K/\bar{\lambda}, \; \omega_s/\bar{\lambda}\}$

Proof: As a preliminary step to determine global stability of the closed-loop system, we select a Lyapunov function candidate, $V: (t, e_s, \tilde{c}) \in R^+ \times R^n \times R^{m+1} \rightarrow R^+$, which is a scalar and differentiable function (at least C^1 function);

$$V = {}^1\!/_2 \, \bar{y}^T W \bar{y} \tag{7}$$

where $\bar{y}^T = [e_s^T \tilde{c}^T]$ and $W = \text{Block diag}[M, P_s^{-1}]$. Observing that M and P_s are all positive-definite matrices, we have

$${}^1\!/_2 \underline{\lambda} \| \bar{y} \|^2 \le V \le {}^1\!/_2 \bar{\lambda} \| \bar{y} \|^2$$

where $\underline{\lambda} = \lambda_{\min}(W) > 0$ and $\bar{\lambda} = \lambda_{\max}(W)$. Thus, V is a positive-definite function. Under these conditions, the derivative $\dot{V}$ along the trajectories of the system can be easily computed as

$$\dot{V} = e_s^T M \dot{e}_s + {}^1\!/_2 e_s^T \dot{M} e_s + \tilde{c}^T P_s^{-1} \dot{\tilde{c}}$$

Suppose that the adaptation law (5) are chosen. Then it is not difficult to show that

$$\dot{V} = e_s^T \{ -C e_s - (T_u + T_s) - K e_s - \frac{\hat{\Phi}_s^2 e_s}{\| e_s \| \hat{\Phi}_s + \eta} \} + {}^1\!/_2 e_s^T \dot{M} e_s$$

$$-\tilde{c}^T(U_s^T\|e_s\|-\omega_s\hat{c}) \tag{8}$$

in which we make use of the fact that $\dot{\tilde{c}}=-\dot{\hat{c}}$ (provided $\dot{c}=0$). By invoking [P3] in paper (I), we immediately get

$$\dot{V}\le -e_s^TKe_s+\|e_s\|U_sc - \frac{\hat{\Phi}_s^2\|e_s\|^2}{\|e_s\|\hat{\Phi}_s+\eta} - \tilde{c}^TU_s^T\|e_s\| + \tilde{c}^T\omega_s\hat{c}$$

which in turn yields

$$\begin{aligned}\dot{V}&\le -e_s^TKe_s+\|e_s\|U_sc-(c^T-\tilde{c}^T)U_s^T \\ &\quad \|e_s\| - \frac{\hat{\Phi}_s^2\|e_s\|^2}{\|e_s\|\hat{\Phi}_s+\eta}+\tilde{c}^T\omega_s(c-\tilde{c}) \\ &\le -e_s^TKe_s+\|e_s\|\hat{\Phi}_s - \frac{\hat{\Phi}_s^2\|e_s\|^2}{\|e_s\|\hat{\Phi}_s+\eta} \\ &\quad -\omega_s\|\tilde{c}\|^2+\tilde{c}^T\omega_sc \\ &\le -e_s^TKe_s+\frac{\eta\hat{\Phi}_s\|e_s\|}{\|e_s\|\hat{\Phi}_s+\eta}-\omega_s\|\tilde{c}\|^2 \\ &\quad +\omega_s\|\tilde{c}\|\|c\|\end{aligned} \tag{9}$$

Note that

$$-\omega_s\|\tilde{c}\|^2+\omega_s\|\tilde{c}\|\|c\|\le-(\omega_s/2)\|\tilde{c}\|^2+(\omega_s/2)\|c\|^2$$

it follows by direct calculation that

$$\dot{V}\le -K\|e_s\|^2-(\omega_s/2)\|\tilde{c}\|^2+\eta +(\omega_s/2)\|c\|^2 \tag{10}$$

Let

$\xi=(\omega_s/2)\|c\|^2$ and $\xi_0=\min\{2K/\bar{\lambda},\ \omega_s/\bar{\lambda}\}$ where $\xi(\in R^+)$ and $\xi_0(>0)$. The differential inequality can be written simply

$$\dot{V}\le -\xi_0V+\eta+\xi \tag{11}$$

In particular, if $\eta\neq 0(\sigma>0,\ \kappa=0)$, i.e., a boundary layer (or saturation-type) controller, then, for $V_f=(\sigma+\xi)/\xi_0$, one simply verifies that $\dot{V}<0$, which is negative-definite, if and only if $V>V_f$ (or $V\subset H^c$), $\forall(e_s,\ \tilde{c})\in H^c$. Consequently, the solution to (11) is given by

$$V\le\exp(-\xi_0t)[V_0-(\sigma+\xi)/\xi_0] +(\sigma+\xi)/\xi_0,\ t\ge 0 \tag{12}$$

where $V_0=V_{t=0}(\bullet)$. The corresponding ultimate bound is given by

$$0\le\lim_{t\to\infty}V=\inf_t V=V_f=(\sigma+\xi)/\xi_0\le V_0<\infty$$

If $\eta\neq 0(\sigma>0,\ \kappa>0)$, then the boundedness of V can be obtained by, for all $t\ge 0$

$$V\begin{cases}\le\exp(-\xi_0t)[V_0-\sigma/(\xi_0-\kappa)-\xi/\xi_0]+\exp(-\kappa t)\sigma/(\xi_0-\kappa)+\xi/\xi_0,\ \xi_0\neq\kappa\\ \le\exp(-\xi_0t)[V_0-\xi]+\sigma t\exp(-\xi_0t)+\xi,\ \xi_0=\kappa\end{cases} \tag{13}$$

Then the ultimate bounds are given by

$$0\le\lim_{t\to\infty}V=\inf_t V=V_f=(\sigma+\xi)/\xi_0\le V_0,\ \xi_0\neq\kappa$$

and $0\le\lim_{t\to\infty}V=\inf_t V=V_f=\xi,\ \xi_0=\kappa$

It should be noted that V converge to the compact set H exponentially with the rate of convergence depending on the values of ξ_0 and κ. Moreover, from (12) and (13), the norm bounds of the joint tracking errors can be estimated as follows:
In the case that $\eta\neq 0(\sigma>0,\ \kappa=0)$, we have

$$\|e_s\|\le\sqrt{2/\underline{\lambda}}\{\exp(-\xi_0t)[V_0-(\sigma+\xi)/\xi_0] +(\sigma+\xi)/\xi_0\}^{1/2} \tag{14}$$

Similarly, in case of $\eta\neq 0(\sigma>0,\ \kappa>0)$, we further obtain

$$\|e_s\|\le\begin{cases}\sqrt{2/\underline{\lambda}}\left[\exp(-\xi_0t)\left(V_0-\frac{\sigma}{\xi_0-\kappa}-\frac{\xi}{\xi_0}\right)+\exp(-\kappa t)\left(\frac{\sigma}{\xi_0-\kappa}+\frac{\xi}{\xi_0}\right)\right]^{1/2},\ \xi_0\neq\kappa\\ \sqrt{2/\underline{\lambda}}\,[\exp(-\xi_0t)(V_0-\xi)+\sigma t\exp(-\xi_0t)+\xi]^{1/2},\ \xi_0=\kappa\end{cases} \tag{15}$$

Hence, it is clear from (14) and (15) that the norms of the trajectory tracking errors are attracted into the following compact set as $t\to\infty$:
In the case that $\eta\neq 0(\sigma>0,\ \kappa=0)$, the residual set is

$$\Omega(e_s)=\{e_s\in R^n:\ \|e_s\| \le[2(\sigma+\xi)/\underline{\lambda}\xi]^{1/2}\} \tag{16}$$

And by selecting $\eta\neq 0(\sigma>0,\ \kappa>0)$, it is readily verified that

$$\Omega(e_s)=\begin{cases}e_s\in R^n:\ \|e_s\|\le 2\xi/\underline{\lambda}\xi_0,\ \xi_0\neq\kappa\\ e_s\in R^n:\ \|e_s\|\le 2\xi/\underline{\lambda},\ \xi_0=\kappa\end{cases}$$

which completes the proof. In fact, the UUB results of tracking errors (e_s) in which the radius of the closed balls depend on types of control structures are obtained. The uniform ultimate boundednesses of other signals ($\tilde{c}$) are also guaranteed in similar fashions as in e_s. As a consequence, the UUB results for system responses (e_s, $\tilde{c}$) are established with respect to $\bar{V}_f$. Clearly, all signals in the closed-loop dynamics are finally attracted into the target ball H (also called region of attraction, domain of attraction, or basin) in finite time regardless of significant uncertainties. Furthermore, the size of the tracking errors can be reduced by manipulating the design parameters.

Finally, we remark that: The UUB results of the tracking errors (e_p and $\dot{e}_v$) can be deduced from that of e_s; For the specific value $\eta=0$ (or $\sigma=0$) in (11), it is readily shown that the closed-loop system is also uniformly ultimately bounded; Furthermore, if $\eta=0$ and $\xi=0$, then the global exponential stability result can be obtained as

$$\lim_{t\to\infty} \| e_s(t) \| = 0$$

which accordingly ensures that $\lim_{t\to\infty} \| e_p(t) \| = 0$ and $\lim_{t\to\infty} \| \dot{e}_v(t) \| = 0$, where the region of attraction is the whole space R^n.

4. Numerical Simulations

In this section, we will attempt to uncover essential issues by studying numerical simulations. The control algorithms are simulated on an uncertain two-link robotic arm whose dynamic model and configuration are given in the paper (1). The initial conditions for unknown constants c are given by $c(0)=[0.0\ 0.0\ 0.0]^T$. And some numerical values of the design parameters are selected as

$$K=200,\ \varepsilon=2.0,\ \eta=0.1(\sigma=0.1,\ \kappa=0)$$
$$U_u=\text{diag}(15.0,\ 5.0,\ 5.0)$$
$$\omega_f=2.0(rad/s),\ \text{and}\ \omega_s=0.1$$

All other quantities including the design variables are the same as those used for the nonadaptive case (1). The simulation results are depicted in Figs. 2~6. We also provide simulation result in Fig. 7 to compare the performance of the proposed control laws with that of the PD controller with control gains being the same. It

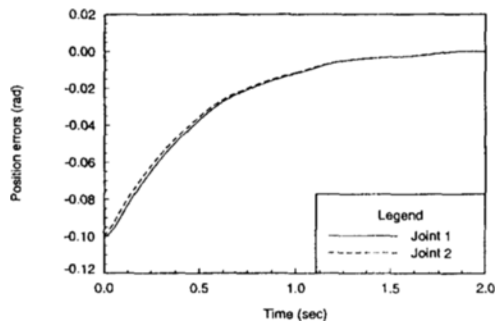

Fig. 2 Joint position tracking errors

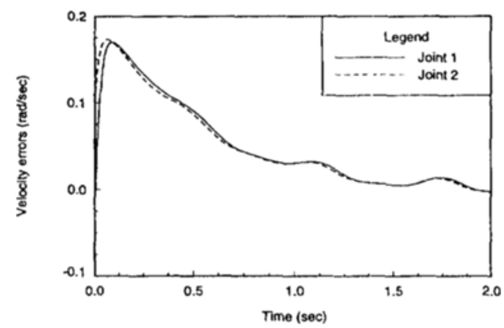

Fig. 3 Joint velocity tracking errors

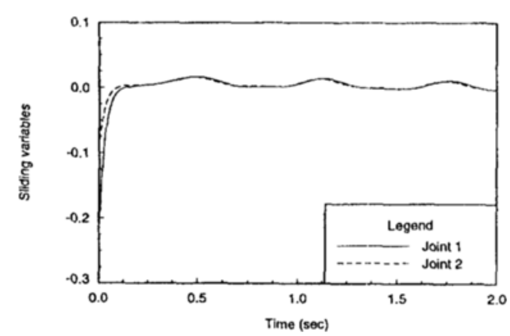

Fig. 4 Sliding surface variables (or joint tracking errors)

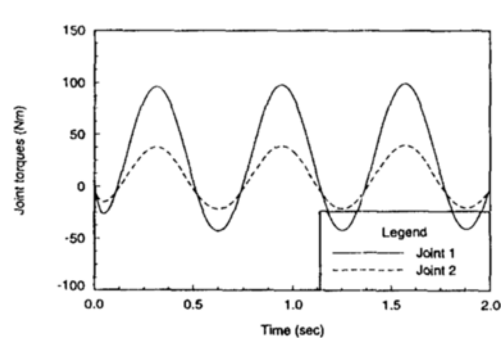

Fig. 5 Control input torques

appears that the closed-loop system in PD control is unstable. As illustrated in simulation results, the adaptive bounds of robust controllers clearly outperforms the PD controller and the nonadaptive version of robust controllers (as in Paper(I)) in terms of the speed of the system responses (including the transient responses as well as the steady-state responses), the joint position tracking errors, the joint velocity tracking errors, and the sliding variable tracking errors. However, more intensive computations are required to update the uncertainty bounds in the adaptive control part ($\bar{\bar{T}}_n$). As a matter of fact, the robust controllers (as in Paper(I)) are simple to implement in practice compared to the controllers proposed in Paper(II) and other centralized adaptive controllers.

As final remarks, we have to state the following results (You, 1994c) The choices of η (or σ and κ) in (6) affect the transient and overall system responses ; ii) Care should be taken in choosing specific values of η (or σ and κ) to avoid chattering phenomena ; iii) The convergence of system responses, at least guaranteeing the UUB results, can be generally rated from fastest to slowest in order as follow, purely discontinuous controller ($\eta=0$; $\sigma=0$), smooth controller ($\eta\neq0$; $\sigma>0$, $\kappa>0$), and saturation-type controller ($\eta\neq0$; $\sigma>0$, $\kappa=0$).

As shown in Figs. 7~8 of the paper (I), the drawbacks of a discontinuous control law in (6) (that is, as $t\rightarrow\infty$, $\eta\rightarrow0$; $\exp(-\kappa t)\rightarrow0$) are that it causes undesirable phenomena such as chattering associated with excessive control activity and exciting high-frequency unmodelled dynamics. Thus, even if the design parameters (η, σ, and κ) may be selected arbitrarily, the trade-off should be made between practical control energy (or chattering) and better system tracking performances (including the speed of the system responses) to meet the design specifications.

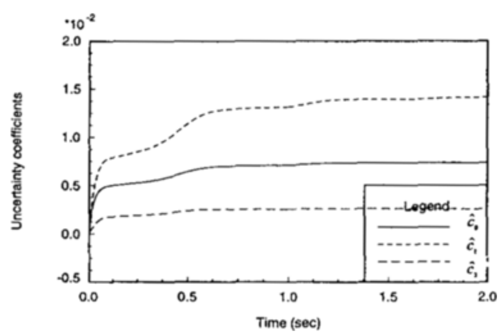

Fig. 6 Estimation of uncertainty bound coefficients

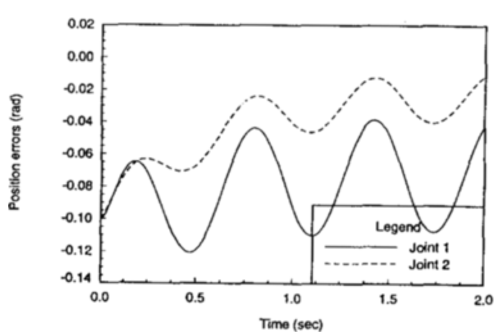

Fig. 7 Joint position tracking errors under PD control law

5. Conclusions

This research has been devoted to presenting dynamic compensation methodology for robust trajectory tracking controls of robot system when its physical model is not completely known. The proposed control scheme consists of two major parts, that is, fully model-based feedforward plus PD compensation and adaptive versions of robust controllers. The robust control synthesis adopted is based on a deterministic approach in which the controllers can be implemented in a decentralized manner. By Lyapunov's second method, the stability and robustness issues of the closed-loop system have been investigated extensively and rigorously. Summarizing, we state the following results : (i) the joint accelerations are not required in the control laws ; (ii) the control laws do not require the exact information about the system parameters and the dynamic models ; (iii) torque computations in the model-based portion can be calculated off-line before control ; (iv) if the possible bounds of uncertainties are assumed to be known, the nonadaptive versions of robust controllers (Paper(I)) are designed ; if no information on these bounds is available, the adaptive versions of the robust controllers are presented to directly estimate the unknown bounds (as in Paper(II)) ; (v) the adaptive bounds of robust controls can cope with any higher-order uncertainties in the system ; (vi) it is shown that the proposed control laws can guarantee at least the UUB results of all signals under any higher-order uncertainties ; (vii) the robot manipulator is made to follow a class of desired trajectories fast while maintaining good tracking performance. Finally, the proposed control approaches encompass earlier results on the model-based feedforward compensations with the VS-type robust controllers as a special case.

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